Short-Term Hydrothermal Coordination by Augmented Lagrangean Relaxation: a new Multiplier Updating

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Abstract

The Augmented Lagrangean Relaxation (ALR) method is one of the most powerful techniques to solve the Short-Term Hydrothermal Coordination (STHC) problem. A crucial step when using the ALR method is the updating of the multipliers. In this paper we present a new multiplier updating procedure: the Gradient with Radar Step (GRS) method. The method has been successfully tested by solving medium-scale examples of the STHC problem.

Keywords. Augmented Lagrangean Relaxation (ALR) method, Classical Lagrangean Relaxation (CLR) method, Gradient with Radar Step (GRS) method, Short-term Hydrothermal Coordination (STHC) problem, Variable Duplication (VD) method, Block Coordinate Descent (BCD) method.

1 Introduction

The problem dealt with below is called the Short-Term Hydrothermal Coordination (SHTC) problem. The objective of this problem is the optimization of electrical production and distribution, considering a short-term planning horizon (from one day to one week). Hydraulic and thermal plants must be coordinated in order to satisfy the customer demand of electricity at the minimum cost and with a reliable service.

The model for the STHC problem presented here considers the thermal system, the hydraulic system and the distribution network. The starting point

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is the paper by Batut and Renaud [1] and therefore we use Variable Duplication plus the Augmented Lagrangean Relaxation (ALR) method. The method used by Batut and Renaud is improved theoretically and practically. From the theoretical point of view, the conservative Auxiliary Problem Principle (see [1]) is replaced by the Block Coordinate Descent Method (see [2]) that shows to be faster. From the practical point of view, an effective software package designed to solve the Optimum Short-Term Hydrothermal Scheduling Problem, (see [6]), is incorporated in order to speed up the whole algorithm. A new multiplier updating method is introduced and compared with the classical multiplier method [2].

This paper is divided into the following sections:

1. Introduction.
2. Formulation of the problem.
3. Modeling the STHC problem.
4. Solution algorithm.
5. The Gradient with Radar Step (GRS) method.
6. Solving the STHC problem.
7. Computational tests.
8. Conclusions.
9. References.

2 Formulation of the problem

The optimization problems here considered are of the following type (P1):

$$\left.\begin{array}{ll} \min & f(x) = C_{htd}(x) + C_m(x) \\ s.t. & x \in \mathcal{D}_{htd} \\ & x \in \mathcal{D}_m \end{array}\right\} \quad (1)$$

Where:

- $\mathcal{D}_{htd}$ represents the domain defined by the constraints that couple the hydraulic, thermal and distribution systems: load constraints, spinning reserve constraints, etc.
- $\mathcal{D}_m$ represents the domain of the management for the thermal units.
- $C_{htd}(x)$ represents the costs associated with $\mathcal{D}_{htd}$.
- $C_m(x)$ represents the costs associated with D_m.

3 Modeling the Short-Term Hydrothermal Coordination (STHC) problem

The general expression of the STHC problem (P1) could be developed in several different ways. The approach adopted in this paper follows the so called *Coupled Model* presented in [6]. This model takes into account the hydroelectric energy generation system together with the thermal system and the transmission network. The variable vector x of the problem (P1) splits into three different vectors, x_H for the variables related with the hydroelectric system (volume, discharges and spillages of each reservoir), x_T for the thermal variables (power output and spinning reserve of each thermal unit), and variables x_E which account for the power flow through the electric transmission network. In the Coupled Model the constraints relating all these variables (domain $\mathcal{D}_{htd}$ of problem (P1)) are expressed through a network flow model with side constraints:

$$
\begin{aligned}
A_{HTT}\begin{pmatrix} x_H \\ x_T \\ x_E \end{pmatrix} &= b_{HTT} && (2)\\
h(x_H, x_E) &= 0 && (3)\\
T_{ISR}x_T &\geq b_{ISR} && (4)\\
T_{DSR}x_T &\geq b_{DSR} && (5)\\
T_{KVL}x_E &= 0 && (6)\\
\underline{x}_H \leq x_H &\leq \bar{x}_H && (7)\\
\underline{x}_T \leq x_T &\leq \bar{x}_T && (8)\\
\underline{x}_E \leq x_E &\leq \bar{x}_E && (9)
\end{aligned}
$$

where

- (2): are the network constraints associated with the so-called *Hydro-Thermal-Transmission Extended Network* (HTTEN). The HTTEN integrates the replicated hydro network, which accounts for the time and space coupling between the reservoirs of the river basin, the *thermal equivalent network* which define the relation between the power output and the spinning reserve level of each thermal unit, and the transmission network, which formulates the conservation of the power flow at the busses of the transmission system.

- (3): these nonlinear side constraints defines the injection of the hydroelectric generation (a nonlinear function of the variables x_H) into the appropriate busses of the transmission network. As it will be explained later, the solution procedure will be based on a successive linearization of these constraints.

- (4),(5): These two sets of linear side constraints impose the satisfaction of the incremental and decremental spinning reserve requirements of the whole system.

- (6): this last set of linear side constraints is the formulation of the Kirchoff Voltage Law. These constraints, together with the power flow conservation equations formulated in (2), represent a dc approach to the transmission network.

- (7),(8),(9): upper and lower bounds to the variables.

The formulation of the domain $\mathcal{D}_{htd}$ as a network flow problem with side constraints allows one the use of specialized network optimization codes. Also, the flexibility of this model is such that other relevant system constraints can be easily added, for instance, security constraints and emission constraints (See [3]).

The thermal management domain $\mathcal{D}_m$ of the problem (P1) deals with the constraints of the unit commitment problem, namely, the minimum down time and maximum up time of the thermal units.

The first term of the objective function of (P1), $C_{htd}(x)$, represents the 50 % of the cost of the fuel consumption of the thermal units, and it is modeled as a quadratic function of the power output of each thermal unit. This term could also include an estimation of the cost of the power losses through a quadratic function of some of the variables x_E. The second part, $C_m(x)$, includes the remaining 50 % of the fuel cost, the start-up and shut-down costs of the thermal units, and depends only on the thermal variables x_T.

4 Solution algorithm

Nowadays the Lagrangean Relaxation (LR) method is the most widespread procedure to solve the STHC problem. The initial Classical Lagrangean Relaxation (CLR) method was improved by the Augmented Lagrangean Relaxation (ALR) method during the past decade (see [1,2,8,9]), although recent advances in the multiplier updating for the CLR method (cutting plane, bundle methods, etc.) have brought this classical method back into fashion (see [10,11,12]).

Some advantages of the ALR method are (see [1,2,8]):

- In the ALR and in the CLR method we maximize a concave function: the dual function.

- The ALR method allows us to obtain a saddle-point even in cases where the CLR method presents a duality gap. The solution of the STHC problem by the CLR method usually yields an unfeasible primal solution x_k due to the duality gap, whereas in the ALR method a solution of the dual problem provides a feasible primal solution.

- The ALR method is a mixture of the CLR method with the penalty method. On the one hand, the penalty term avoids the typical oscillations of the CLR method. On the other hand, the Lagrangean term avoids the typical ill conditioning of the penalty method, which usually requires large penalty terms.
- Using the CLR method, the differentiability of the dual function cannot be ensured and therefore nondifferentiable methods must be applied in the CLR method. This difficulty can be overcome if an augmented Lagrangean is used, since the dual function q_c is differentiable for an appropriate c. Thus, the multipliers can be updated using "large steps".

The weaknesses of the ALR method are:

- The quadratic terms introduced by the augmented Lagrangean are not separable. If we want to solve a problem by decomposition, some methods, such as the Auxiliary Problem Principle, (see [4]), or, as in our case, the Block Coordinate Descent method, (see [2]), must be used. However, the CLR method gives a separable Lagrangean.
- The multiplier updating is done in a heuristic way, (see [2]), that needs to be tuned.

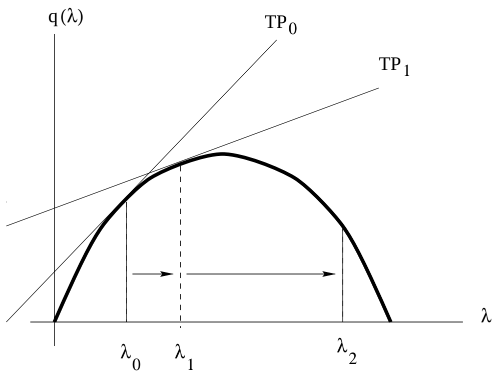
  $$\lambda_{k+1} = \lambda_k + c_k \nabla q_c(\lambda_k) \tag{10}$$

  We introduce a new multiplier updating procedure that completely overcomes this difficulty: the Gradient with Radar Step (GRS) method.

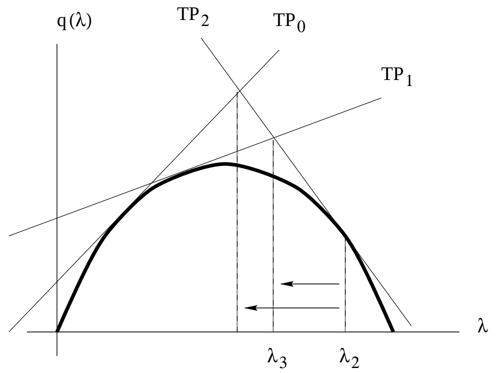
5 The Gradient with Radar Step (GRS) method

The objective of this method is to maximize a differentiable and concave function $q(\lambda)$ without constraints. This method uses the same information as the cutting plane method but in a different way. The tangent planes obtained in the course of the optimization give us a first order approximation of $q(\lambda)$ and the cutting plane method directly optimizes these successive approximations of $q(\lambda)$. Alternatively, the GRS method uses the approximation to $q(\lambda)$ in order to compute the step length for an ascent direction (such as the gradient). Although convergence of the GRS method has not yet been proved, experience shows a very good behavior regarding convergence.

The geometrical intuition of the GRS method is displayed in the pictures below. In the first situation, given that no previous stopping tangent planes exist, move λ_n one *step* in the gradient direction:



In the second situation, given that there is at least one previous stopping tangent plane, move λ_n up to the first stopping tangent plane in the gradient direction. In the picture below, λ_3 is given by the intersection of TP_2 with TP_1, the first stopping plane.



A rough draft of the GRS method is given below:

Algorithm GRS

- * [Method.] Gradient with Radar Step.
- * [Objective.] It solves the following problem, where $q(\lambda)$ is a concave and differentiable function and numerical line-search cannot be applied (e.g., the dual function):

$$\max_{\lambda \in R^m} \quad q(\lambda) \tag{11}$$

- * [Input.] λ_0 initial point.
- * [Output.] λ^* optimizer of $q(\lambda)$.

Step 0 [Initialize.] Set $n = 0$.

Step 1 [Compute the gradient vector.] Compute $g_n := \nabla q(\lambda_n)$. Let TP_n be the first order approximation of $q(\lambda)$ at the point $(\lambda_n, q(\lambda_n))$ i.e., TP_n is the tangent plane defined by g_n. Store the tangent plane TP_n.

Step 2 [Check the stopping criterion.] If $g_n = 0$ then stop. λ_n is the optimizer of $q(\lambda)$.

Step 3 [Compute the step length]. Move over TP_n in such a way that λ_{n+1} moves along the straight line $\lambda_{n+1} = \lambda_n + \beta \cdot g_n$, with $\beta > 0$. Keep moving up to the first stopping tangent plane TP_k with $k < n$. This means we stop the advance of λ_{n+1} for a value of β, say β_n. If no such stopping plane exists set $\beta_n = c$, the penalty parameter.

Step 4 [Truncate the step length by the multiplier method step.] Set $\tilde{\beta}_n = \min\{\beta_n, c\}$.

Step 5 Compute $\lambda_{n+1} = \lambda_n + \tilde{\beta}_n g_n$. Set $n = n + 1$ and go back to Step 1.

Proposition GRS step

* [Definitions.]

n	GRS iteration index.
λ_n	current iterate.
g_n	$:= \nabla q(\lambda_n)$.
q_n	$:= q(\lambda_n)$.
TP_k	$\equiv y_k(\lambda) = q_k + g_k(\lambda - \lambda_k)$ tangent planes at the point (λ_k, q_k) $\quad (k = 0, \cdots, n-1)$.
$\lambda_{n+1}(\beta)$	$:= \lambda_n + \beta \cdot g_n$ line defined by the point λ_n and the vector g_n.
$(\lambda_{n,k}, y_{n,k})$	intersection point of the line $y_n(\lambda_{n+1}(\beta))$ with the tangent plane TP_k.
$\beta_{n,k}$	step length from λ_n to $\lambda_{n,k}$, i.e., $\lambda_{n,k} = \lambda_n + \beta_{n,k} \cdot g_n$.
β_n	step length of the GRS method, i.e., $\lambda_{n+1} = \lambda_n + \beta_n \cdot g_n$.

* [Hypothesis.] $q(\lambda)$ is a concave and differentiable function.

* [Thesis.] The step length β_n of the previous algorithm GRS can be computed as follows. First, compute

$$\beta_{n,k} := \frac{q_k - q_n + (\lambda_n - \lambda_k)' g_k}{(g_n - g_k)' g_n} \qquad k = 0, \cdots, n-1 \tag{12}$$

and then,

$$\beta_n := \min\{\beta_{n,k} : \quad \beta_{n,k} > 0 \quad k = 0, \cdots, n-1\} \tag{13}$$

A proof of the proposition above can be found in [13].

As we will see en the next section, in the resolution of the STHC problem we only relax the equality constraint of the primal problem $x = \tilde{x}$ in order to get a dual problem with no constraints upon the dual variables (multipliers). Therefore, in our case, the objective of the ALR method is to maximize a differentiable and concave function (the dual function $q_c(\lambda)$) without constraints, characteristics fully coincident with the requirements of the GRS method. Note that in the case of the dual function a classical line-search procedure would be computationally too expensive. The main features of the GRS method are:

- No parameter tuning needs to be done.
- The GRS method, unlike the classical multiplier method, is based on a direct knowledge of the dual function given that it uses a first order approximation of the dual function.
- The information used by the GRS method is computationally very cheap in the Lagrangean framework because the gradient of the dual function is given by the unfeasibility of the relaxed constraint, i.e., $\nabla q_c(\lambda_k) = h(x_k)$, where $h(x)$ is the relaxed equality constraint (in the STHC problem $\nabla q_c(\lambda_k) = x_k - \widetilde{x}_k$). The consequence of this almost free knowledge of the dual function is a computationally efficient and fast updating method.

6 Solving the Short-Term Hydrothermal Coordination (STHC) problem

We follow and improve the algorithm described by Batut and Renaud [1] in the solution of the STHC problem (1). This algorithm uses the Augmented Lagrangean Relaxation (ALR) and Variable Duplication (VD) methods, and previous software used to solve the dispatching problem and the optimal power flow can also be incorporated. To deal with the inseparable Lagrangean, instead of the Auxiliary Problem Principle used by Batut and Renaud we use the Block Coordinate Descent method, that shows to be faster (see [13]).

- First, the Variable Duplication method consists of exactly what the name suggests: the vector of the variables x is duplicated, resulting in $\widetilde{x}$, and then the equality constraint $x = \widetilde{x}$ is added. Thus, we solve the following transformation of (1), which is equivalent to problem (P1):

$$\left.\begin{array}{ll} \min & f(x,\widetilde{x}) = C_{htd}(x) + C_m(\widetilde{x}) \\ s.t. & x \in \mathcal{D}_{htd} \\ & \widetilde{x} \in \mathcal{D}_m \\ & x = \widetilde{x} \end{array}\right\} \qquad (14)$$

- Second, the induced dual problem is:

$$\max_{\lambda \in R^n} \left\{ \begin{array}{ll} \min & L_c(x,\widetilde{x},\lambda) \\ s.t. & x \in \mathcal{D}_{htd} \\ & \widetilde{x} \in \mathcal{D}_m \end{array} \right\}, \tag{15}$$

where

$$L_c(x,\widetilde{x},\lambda) := C_{htd}(x) + C_m(\widetilde{x}) + \lambda'(x-\widetilde{x}) + c\|x-\widetilde{x}\|^2 \tag{16}$$

- Third, L_c is not a separable function, thus in order to minimize it in a separable way we first do some manipulations.

$$\begin{aligned} L_c(x,\widetilde{x},\lambda) &= C_{htd}(x) + C_m(\widetilde{x}) \\ &\quad +\lambda' x - \lambda'\widetilde{x} + \tfrac{c}{2}\|x-\widetilde{x}\|^2 + \tfrac{c}{2}\|x-\widetilde{x}\|^2 = \\ &= (C_{htd}(x) + \lambda' x + \tfrac{c}{2}\|x-\widetilde{x}\|^2) \\ &\quad + (C_m(\widetilde{x}) - \lambda'\widetilde{x} + \tfrac{c}{2}\|x-\widetilde{x}\|^2) \end{aligned} \tag{17}$$

As we can see in (17) L_c consists of two almost separable members in x and $\widetilde{x}$ except for the quadratic term $\|x-\widetilde{x}\|^2$. In the frame of the Block Coordinated Descent (BCD) method, we split L_c into two functions: one, $L_c^n(x,\lambda_n)$, to be minimized in the domain $\mathcal{D}_{htd}$, and the other $\widetilde{L}_c^n(\widetilde{x},\lambda_n)$, to be minimized in $\mathcal{D}_m$ by fixing one of the vectors x or $\widetilde{x}$ each time. More precisely,

$$L_c^n(x,\lambda_n) := C_{htd}(x) + \lambda_n' x + \frac{c}{2}\|x-\widetilde{x}_n\|^2, \tag{18}$$

$$\widetilde{L}_c^n(\widetilde{x},\lambda_n) := C_m(\widetilde{x}) - \lambda_n'\widetilde{x} + \frac{c}{2}\|x_{n+1}-\widetilde{x}\|^2, \tag{19}$$

where $\widetilde{x}_n$ and x_{n+1} are iterates of the decision variables $\widetilde{x}$ and x respectively. Then in the Augmented Lagrangean Relaxation (ALR) algorithm that solves the dual problem (15) the minimization of the augmented Lagrangean over $\mathcal{D}_{htd}$ and $\mathcal{D}_m$ is replaced by two subproblems, one in each domain:

- Hydrothermal subproblem.

$$\left. \begin{array}{ll} \min & L_c^n(x,\lambda_n) \\ s.t. & x \in \mathcal{D}_{htd} \end{array} \right\} \tag{20}$$

- Thermal subproblem.

$$\left.\begin{array}{ll} \min & \widetilde{L}_c^n(\widetilde{x}, \lambda_n) \\ s.t. & \widetilde{x} \in \mathcal{D}_m \end{array}\right\} \qquad (21)$$

Algorithm MACH

Let us suppose we have the following information available: an initial estimate of the Lagrange multipliers λ_0; a penalty parameter c; a positive integer K, which serves as an upper bound to the number of iterations of the Block Coordinated Descent at each minimization of the aumented Lagrangean; a positive integer N, which serves as an upper bound to the number of Lagrange multiplier updates; an initial point x_0^0 of the domain $\mathcal{D}_{htd}$ and an initial point $\widetilde{x}_0^0$ of the domain $\mathcal{D}_m$; the numerical tolerances ϵ_1, ϵ_2, $\epsilon_3 > 0$. Then the algorithm proposed, called MACH (from *"Modelo Acoplado de Coordinación Hidrotérmica"*), is:

- **MACH0.-** [Initialize.] Set $n = 0$ and $k = 0$.
- **MACH1.-** [Check the stopping criterion.] If the gradient of the dual function is small enough, i.e., $\|x_n^0 - \widetilde{x}_n^0\| < \epsilon_1$ then stop. The algorithm terminates with $(x_n^0, \widetilde{x}_n^0, \lambda_n)$ as a solution. If $n > N$ the algorithm has failed.
- **MACH2.-** [Solve the hydrothermal subproblem.] With x_n^k as an initial point and $\widetilde{x}_n^k$ as a fixed vector, execute a procedure to solve the following subproblem:

$$\min_{x \epsilon \mathcal{D}_{htd}} L_c^n(x) = \min_{x \epsilon \mathcal{D}_{htd}} \left[C_{htd}(x) + \lambda_n' x + \frac{c}{2}\|x - \widetilde{x}_n^k\|^2 \right] \qquad (22)$$

  including security measures to deal with unboundedness. Let x_n^{k+1} be the calculated solution.
- **MACH3.-** [Solve the thermal subproblem.] With $\widetilde{x}_n^k$ as an initial point and x_n^{k+1} as a fixed vector, execute a procedure to solve the following subproblem:

$$\min_{\widetilde{x} \epsilon \mathcal{D}_m} \widetilde{L}_c^n(\widetilde{x}) = \min_{\widetilde{x} \epsilon \mathcal{D}_m} \left[C_m(\widetilde{x}) - \lambda_n' \widetilde{x} + \frac{c}{2}\|x_n^{k+1} - \widetilde{x}\|^2 \right] \qquad (23)$$

  including security measures to deal with unboundedness. Let $\widetilde{x}_n^{k+1}$ be a solution.
- **MACH4.-** [Repeat MACH2 and MACH3 until no progress can be done.]

If $(k \geq K)$ then go to step 5.

If

$$\|x_n^{k+1} - x_n^k\| \quad > \quad \epsilon_2 \qquad or \qquad \|\tilde{x}_n^{k+1} - \tilde{x}_n^k\| \quad > \quad \epsilon_3 \tag{24}$$

then set $k = k + 1$ and go back to MACH2.

- **MACH5.-** [Dual variable updating.] Update the multiplier estimates using the GRS method or the multiplier method.

$$\lambda_{n+1} = \lambda_n + \beta_n(x_n^{k+1} - \tilde{x}_n^{k+1}) \tag{25}$$

- **MACH6.-** [Update the iteration counts.]

  Set $x_{n+1}^0 = x_n^{k+1}$, $\tilde{x}_{n+1}^0 = \tilde{x}_n^{k+1}$, $n = n + 1$ and $k = 0$. Go back to MACH1.

One of the advantages of the Variable Duplication framework is the possibility of incorporating preexisting software. In step MACH2 the minimization of the augmented Lagrangean subject to the constraints (2) to (9) is needed. This is a nonlinear network flow problem with side constraints which can be solved either with general purpose optimization packages or with specialized procedures. The implementation reported in this paper is based on the specialized code NOXCB (see [5]). This code implements an active set method which exploits the network structure through primal partitioning techniques, (see [7]), to solve the nonlinear network problem with linear side constraints. To handle the nonlinear constraint (3) a successive linearization method presented in [6] is used. In this method, a sequence of subproblems are solved in which the nonlinear constraints (3) are linearized over the optimal solution of the previous subproblem. The linearizations stop when a given convergence criterion is reached. Furthermore, in the future this framework will allow the incorporation of new packages, for example Interior Point based software to solve step MACH2.

In step MACH3 a classical forward dynamic programming procedure has been implemented. In our opinion, the characteristics of the subproblem (21) (binary variables plus separability) make the dynamic programming procedure one of the best options.

7 Computational tests

So far we have tested the MACH package, considering the hydraulic and thermal systems without the distribution network, although the software developed can incorporate the distributing network. The CPU times correspond to a Sun/Ultra2 2200 workstation with 200 MHz clock, 256 Mbytes of main memory, 68 Mflops Linpack, 14.7 Specfp95 and 7.8 Speccint95.

In the table below we present 6 instances of the STHC problem that range from 24 to 1848 binary variables. We consider examples which range in size from very small (6 intervals, 2 reservoirs and 4 thermal units) up to medium (168 intervals, 4 reservoirs and 11 thermal units). The parameter K (upper bound to the number of iterations of the Block Coordinated Descent method at each minimization of the augmented Lagrangean) has been set equal to 0. The numerical tolerances, ϵ_1, ϵ_2 and ϵ_3, have been set equal to 10^{-4}. The penalty parameter c used in all cases has been 10.

Table 1: Description of a sample of STHC instances.

Case	n_i	n_r	n_t	n_{con}	n_{bin}
0400601	6	2	4	138	24
0404801	48	2	4	1104	192
0704801	48	2	7	1680	336
0216801	168	4	2	3360	336
0716801	168	4	7	6720	1176
1116805	168	4	11	9408	1848

Legend:

Case Label of the problem case.

n_i Number of intervals (1 interval = 1 hour).

n_r Number of reservoirs.

n_t Number of thermal units.

n_{con} Number of continuous variables.

n_{bin} Number of binary variables (on/off variables).

The results obtained using the MACH package with the multiplier method are shown in table 2. We used a constant penalty parameter version ($c_k = c$ in equation (10)).

Table 2: Solution using the multiplier method.

Case	iter	t	cost	$\%inf$
0400601	9	9.3	13.6	$1.0 \cdot 10^{-4}$
0404801	6	7.7	1.0	$1.2 \cdot 10^{-6}$
0704801	6	7.6	1.1	$1.3 \cdot 10^{-6}$
0216801	2	26.1	4.6	$4.0 \cdot 10^{-4}$
0716801	*	2700	*	*
1116805	4	311	85.8	$9.0 \cdot 10^{-4}$

Legend:

iter Number of multiplier updating iterations.

t CPU time (in seconds).

cost Optimal cost (in millions of pesetas).

$\%inf$ Relative unfeasibility in %.

If $\|x - \tilde{x}\|_\infty := \max\{|x_j - \tilde{x}_j| : j = 1, \cdots, n\} =: |x_0 - \tilde{x}_0|$ then $\%inf := 100 \frac{\|x-\tilde{x}\|_\infty}{0.5 \cdot (|x_0| + |\tilde{x}_0|)}$

* Data not available.

The results obtained using the MACH package with the GRS method are shown in table 3.

Table 3: Solution using the GRS method.

Case	iter	t	cost	$\%inf$
0400601	10	10.3	13.6	$9.8 \cdot 10^{-5}$
0404801	6	8.7	1.0	$3.2 \cdot 10^{-6}$
0704801	7	8.5	1.1	$9.6 \cdot 10^{-8}$
0216801	7	32.8	4.5	$4.0 \cdot 10^{-8}$
0716801	10	355.1	3.3	$4.0 \cdot 10^{-8}$
1116805	6	370.3	85.8	$2.0 \cdot 10^{-5}$

A sample of the performance of the MACH package is given in the tables 2 and 3. The main points of the tables are:

1. Usually, both methods reach an optimum with 10 or fewer multiplier updates against the 50 or more reported by Batut and Renaud (see [1]). In our opinion, this is due to the use of the Block Coordinate Descent method instead of the conservative Auxiliary Problem Principle method (see [13]). In consequence the CPU time required to

solve the UCCT problem falls drastically in such a way that problems with more than 1000 binary variables (cases 0716801 and 1116805) are solved within 6 minutes.

2. Unlike the Classical Lagrangen Relaxation, this method yields ready-to-use feasible solutions since the relative unfeasibility (% inf) is almost null. Note that a null relative unfeasibility implies no duality gap.

3. The performance of the compared multiplier updating methods is very similar. On the one hand, the classical multiplier method usually reaches an optimum with one or two iterations less than the GRS method. Furthermore, the multiplier method is computationally cheaper than the GRS method which needs to compute n candidates for the step length β_n at each iteration n. On the other hand, we can find examples, as the case 0716801, where the GRS method converged to an optimum in 355 seconds, whereas the multiplier method (with constant penalty parameter c) did not give any solution after 45 minutes of CPU. All in all, we can conclude that when solving the STHC problem the new GRS method does not improve, so far, the classical multiplier method but it can be used as an alternative method.

8 Conclusions

The Short-Term Hydrothermal Coordination (STHC) problem has been solved using the Variable Duplication plus Block Coordinated Descent method within the Augmented Lagrangean Relaxation (ALR) framework. A new multiplier updating method, the Gradient with Radar Step (GRS) method, has been designed and implemented. Three main conclusions must be pointed out:

First, the ALR framework designed by Batut and Renaud to solve the STHC notably improves if the Block Coordinated Descent method is used instead of the Auxiliary Problem Principle.

Second, after our computational experience, the new GRS method does not improve, so far, the classical multiplier method but it can be used as an alternative method.

Third, in order to solve the STHC problem the ALR method implemented in the MACH package, represents a competitive alternative to the Classical Lagrangean Relaxation (CLR), mainly due to the ALR method directly gives a primal feasible solution. The CLR obtains unfeasible primal solutions that must be processed by some heuristic procedure in order to reach feasibility.

9 References

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